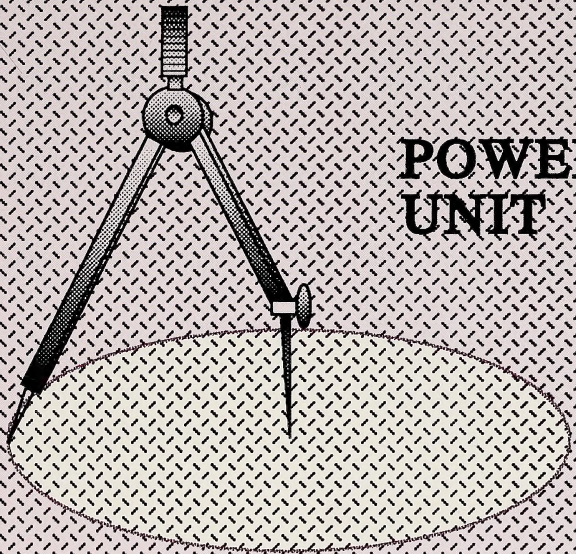
MATHEMATICS 23



POWERS AND RADICALS
UNIT 1

Distance Learning

Alberta
EDUCATION

Welcome

Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

Mathematics 23 Student Module Unit 1 Powers and Radicals Alberta Distance Learning Centre ISBN No. 0-7741-0786-3 *1992

m5####

General Information

This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.

- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.

- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.

- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.

- **Extensions** gives you the opportunity to take the topic one step further.

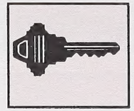
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.

- The **Appendices** include the solutions to **Activities** (**Appendix A**) and any other charts, tables, etc. which may be referred to in the topics (**Appendix B, etc.**).

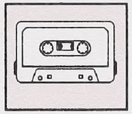
Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

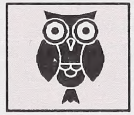
An explanation of what they mean is written beside each visual cue.



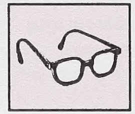
Key Idea
- flagging important ideas



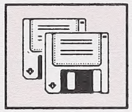
Audiotape
- learning by listening to an audiotape



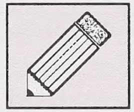
What You Already Know
- reviewing what you already know



Another View
- exploring different perspectives

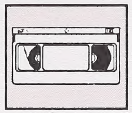


Computer Software
- learning by using computer software

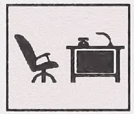


Review
- studying previous concepts

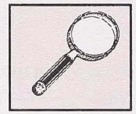
Solutions
- correcting the activities



Videotape
- learning by viewing a videotape

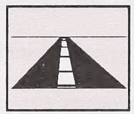


Introduction
- introducing the unit

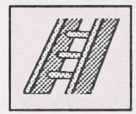


Extra Help
- providing additional study

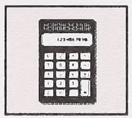
Print Pathway
- choosing a print alternative



What Lies Ahead
- previewing the unit

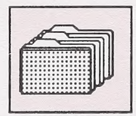


Extensions
- going on with the topic



Calculator
- using your calculator

Exploring the Topic
- actively learning new concepts



What You Have Learned
- summarizing what you have learned

Mathematics 23

Course Overview

Mathematics 23 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 **Powers and Radicals**	**10%**
Unit 2 Algebra	12%
Unit 3 Mathematics of Finance	4%
Unit 4 Linear Relations	12%
Unit 5 Systems of Equations	16%
Unit 6 Geometry	16%
Unit 7 Trigonometry	16%
Unit 8 Statistics	14%
	100%

Unit Assessment

After completing the unit you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal your teacher will determine what this assessment will be. It may be

Unit assignment - 50%
Supervised unit test - 50%

Introduction to Powers and Radicals

This unit covers topics dealing with Powers and Radicals. Each topic contains explanations, examples, and activities to assist you in understanding powers and radicals. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in the **Appendix**. In several cases there is more than one way to do a question.

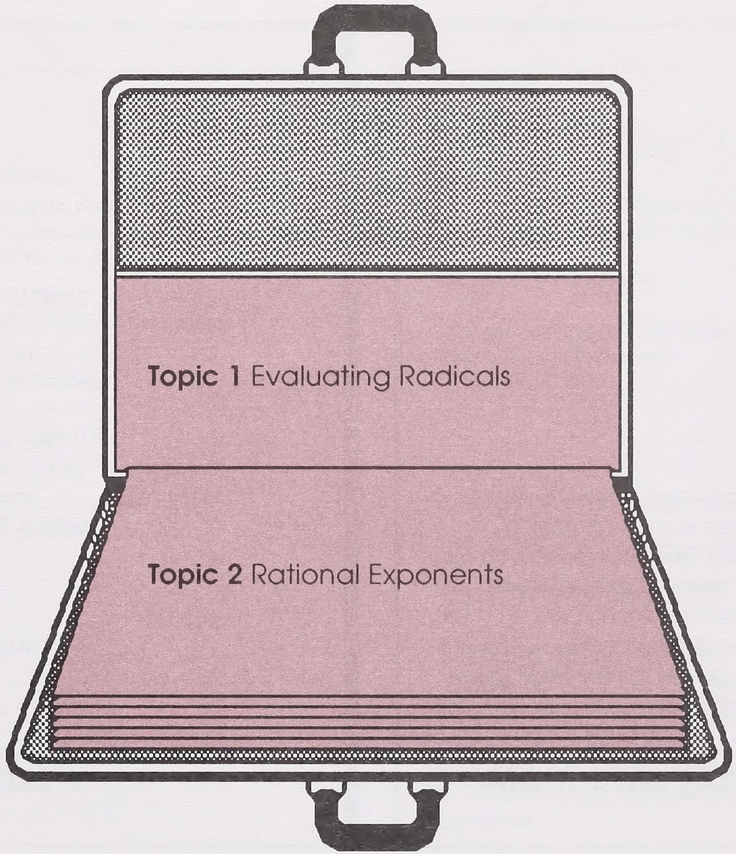
Unit 1 Powers and Radicals

Contents at a Glance

Powers and Radicals

Mathematics uses symbols to describe and communicate "grand" ideas simply and precisely in such a way that anyone who knows the language of mathematics can understand the ideas. In this unit the meaning behind the symbols for powers and radicals will be explored. Exponents were first introduced into mathematics by Rene Descartes, a French mathematician, in 1637. Exponents allow scientists and statisticians to express very large or very small numbers in a more convenient form.

Unit 1 Powers and Radicals



What You Already Know

Refresh your memory!

The following are rules and laws for exponents that you will be required to know to do this unit.

- In the general expression x^m,

  x^m is called a power,

  x is called the base, and

  m is called the exponent.

  For 4^3,

  4^3 is called the power,

  4 is called the base, and

  3 is called the exponent.

- 4^3 means $4 \times 4 \times 4$.

  $$\begin{aligned} 4^3 &= 4 \times 4 \times 4 \\ &= 64 \end{aligned}$$

  The exponent tells you how many 4's are multiplied together.

- Perfect squares (as they will be used in this topic) are numbers that result from the multiplication of identical whole numbers.

  For example, 36 is a perfect square since $6 \times 6 = 36$.

- Product Law: $(x^m)(x^n) = x^{m+n}$

  The following is an example of the Product Law.

  $$\begin{aligned} (2^3)(2^5) &= 2^{3+5} \\ &= 2^8 \end{aligned}$$

- Quotient Law: $\frac{x^m}{x^n} = x^{m-n}$ or $x^m \div x^n = x^{m-n}$

  The following is an example of the Quotient Law.

  $$\begin{aligned} \frac{y^7}{y^3} &= y^{7-3} \\ &= y^4 \end{aligned}$$

- Power Law: $(x^m)^n = x^{mn}$

  The following is an example of the Power Law.

  $$\begin{aligned} (x^3)^4 &= x^{(3)(4)} \\ &= x^{12} \end{aligned}$$

- Power of a Product Law: $(xy)^m = x^m y^m$

  The following is an example of the Power of a Product Law.

  $$(2 \times 7)^2 = 2^2 \times 7^2$$

- Power of a Quotient Law: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

  The following is an example of the Power of a Quotient Law.

  $$\left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3}$$

- $x^0 = 1$, where $x \neq 0$

  The value of any power with a nonzero base and a zero exponent is 1.

- For any nonzero base and natural number *n*,

  $x^{-n} = \frac{1}{x^n}$ and $x^n = \frac{1}{x^{-n}}$, where $x \neq 0$.

Please go to the **Review** to confirm your understanding of the concepts covered in this section.

Review

Do as many questions as you require to ensure you understand the concepts.

1. Identify the base and the exponent for the power 6^3.
2. Find the value of 5^3.
3. Identify the numbers that are perfect squares with whole number factors.

   a. 64 b. 72

   c. 81 d. 49

   e. 25 f. 18

4. Find the square root of the perfect squares in question 3.
5. Write each of the following as a power in simplest form.

   a. $(m^2)(m^2)(m^3)$ b. $a^6 \div a^2$

   c. $(p^4)^3$

6. Simplify using only positive exponents.

a. $\dfrac{\left(4a^2b^3\right)^2}{a^5}$

b. $\dfrac{3x^{-2}y^3}{x^4y^2}$

c. $\dfrac{(5x)^0}{(3x)^{-3}}$

Now go to the **Review Solutions** in the **Appendix**.

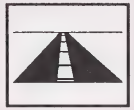
If you had difficulties with these questions, then you may need to review Unit 1 **Number Systems** in Mathematics 13.

Topic 1 Evaluating Radicals

Introduction

Opposites attract! There is a special relationship between addition and subtraction or between multiplication and division. One “undoes” the other. In this topic you will investigate a similar relationship between radicals and exponents.

If you had to make a cubical box of known volume, could you determine the length of the sides of the box? This topic will help you solve such problems.



What Lies Ahead

Throughout the topic you will learn to

1. identify the radicand, index, radical sign, and root in radical expressions
2. evaluate the square root of a perfect square
3. approximate the square root of a number, and check it by using a calculator
4. evaluate the cube root of a perfect cube
5. approximate the cube root of a number, and check it by using a calculator

Now that you know what to expect, turn the page to begin your study of evaluating radicals.

Exploring Topic 1

Activity 1

Identify the radicand, index, radical sign, and root in radical expressions.

You may study this activity by doing either **Part A** or **Part B**, or you may do both. **Part A** covers the activity with an audiotape, while **Part B** covers it through the print mode.

Part A

Audio Activity

Insert the audiotape entitled *Mathematics 23 - Evaluating Radicals* into your tape recorder and follow the instructions on the tape. After you have listened to the tape, complete the questions.

Parts of a Radical

1 Power

5^2

2 Expansion

$5^2 = 5 \times 5$

3 Reverse Procedures

$5^2 = 25$

and

$\sqrt{25} = 5$

4 Symbol for Square Root

$\sqrt[2]{25}$ or $\sqrt{25}$

The $\sqrt{\ }$ means $\sqrt[2]{\ }$.

5 A Radical Expression (A Radical)

$\sqrt{25}$

6 Parts of a Radical

Index $\rightarrow \sqrt[2]{25}$ $\leftarrow$ Radical sign
$\leftarrow$ Radicand

7 Other Radicals

a. $\sqrt[3]{y}$ b. $\sqrt[4]{16n^2}$

8 Example

Consider $\sqrt[3]{8x^6}$.

a. What are the exponents? ____________

b. What is the index? ____________

c. What is the radicand? ____________

d. What is the radical? ____________

9 Complete the Blanks

Expression	Exponent	Radicand	Index	Radical
$\sqrt[5]{7^3}$				
$\sqrt[3]{9m^4}$				

✓ For solutions to the **Audio Activity** , turn to **Activity 1** in the **Appendix, Topic 1**.

Part B

Write the expression 3^2 in expanded form.

$3^2 = 3 \times 3$

If you were asked the question, "What number multiplied by itself equals 9?", you could find the answer in the equation here.

$3 \times 3 = 9$

Therefore, 3 multiplied by itself equals 9.

Mathematicians have developed a set of symbols and notation that make it easy to ask and answer questions of this type. In order to clearly communicate and understand this topic, you must learn these symbols, their names, and the way in which the notation (way of writing things) is used.

"What number multiplied by itself equals 9?" is written as $\sqrt[2]{9}$. The $\sqrt[2]{\ }$ is usually written as $\sqrt{\ }$ because it is the simplest and most commonly used radical sign.

The symbols $\sqrt{9}$ are read as "the square root of 9." You can substitute 3^2 for 9 to get $\sqrt[2]{3^2}$.

Now look at a summary of these terms:

- $\sqrt[n]{x}$ is a radical term.
- x is the radicand; x can be any expression such as $2y + 3$.
- n is the index.
- $\sqrt{\ }$ is the radical sign.

Example 1

Given the expression $\sqrt[5]{2x^3}$, state the following.

- radical

Solution:

$\sqrt[5]{2x^3}$

- index

Solution:

5

- exponent

Solution:

3

- radicand

Solution:

$2x^3$

- radical sign

Solution:

$\sqrt{\ }$

The simplest form of any expression of the form $\sqrt[n]{x}$ is called a **root.**

These roots are denoted in the following way:

$\sqrt[2]{x}$ or $\sqrt{x}$ means the second root or square root of x,

$\sqrt[3]{x}$ means the cube root of x,

$\sqrt[4]{x}$ means the fourth root of x,

$\sqrt[5]{x}$ means the fifth root of x,

.   .

.   .

.   .

$\sqrt[n]{x}$ means the n^{th} root of x.

In all mathematics courses, $\sqrt{\ }$ will mean the positive square root. A mathematical name for the positive square root is the principal square root.

Example 2

Find the cube root of 27.

Solution:

The cube root of 27 is 3 since $3 \times 3 \times 3 = 27$.

Do at least questions 1, 2, and 3. If you want more practice, do the remaining questions. If not, then move on.

1. Given the expression $\sqrt[3]{4^2}$, identify

   a. the index
   b. the exponent
   c. the radicand
   d. the radical

2. What is the square root of 16?

3. What is the cube root of 8?

4. Given the expression $\sqrt[4]{2y^7}$, identify

   a. the exponent
   b. the radicand
   c. the index
   d. the radical

5. What is the square root of 64?

6. What is the cube root of 64?

For solutions to **Activity 1,** turn to the **Appendix, Topic 1**.

You may study the next two activites by listening to the audiotape or by studying the print, or you may do both.

Audio Activity - Calculating Square Roots

Activity 2

Evaluate the square root of a perfect square.

Activity 3

Approximate the square root of a number, and check it by using a calculator.

Your tape (*Mathematics 23 - Evaluating Radicals*) should be cued and ready for this audio activity on calculating square roots. Turn on your tape recorder and follow the instructions on the tape.

Calculating Square Roots

1 Familiar Square Roots

$\sqrt{4}$

$\sqrt{9}$

$\sqrt{16}$

2 Perfect Squares Give Integer Roots

$\sqrt{4}=2$

$\sqrt{9}=3$

$\sqrt{16}=4$

3 Opposite Operations

You know that

$$\sqrt{4}=2$$

is true because

$$2\times 2=4.$$

Similarly,

$$\sqrt{9}=3$$

is true because

$$3\times 3=9.$$

4 Negative Integers Are also Roots

You know $2\times 2=4$.
But, $-2\times -2=+4$ also.
So, the square root of 4 is $+2$ or -2,
and the square root of 9 is $+3$ or -3.

5 The **principal square root** as indicated by the symbol $(\sqrt{\ })$ is the **positive square root.**

i.e. $\sqrt{4}=2$ and $\sqrt{9}=3$ represent principal square roots.

6 Squares and Square Roots

n	n^2	n	n^2
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

7 Square Roots of Other Numbers

What is the square root of 45?

Because 45 lies between 36 and 49, the square root of 45 lies between 6 and 7.

8 A Closer Approximation

$45 - 36 = 9$ and $49 - 36 = 13$.

So, 45 is $\frac{9}{13}$ of the way between 36 and 49.

Hence, $\sqrt{45}$ is $\frac{9}{13}$ of the way between 6 and 7.

$\frac{9}{13} = 0.7$ So, $\sqrt{45} = 6 + 0.7 = 6.7$.

9 Finding Square Roots Using the Calculator

Enter 45.

Press $\boxed{\sqrt{\ }}$

Display = 6.708203982.

10 A Comparison

Find the value of $\sqrt{45}$ (to one decimal).

Approximation = 6.7

Calculator Answer = 6.7

11 Find the Square Roots

$\sqrt{16} =$ ______ $\sqrt{29} =$ ______

$\sqrt{121} =$ ______ $\sqrt{76} =$ ______

12 Square Roots of Variable Expressions

You know $\sqrt{4} = 2$ since $2 \times 2 = 4$.

Similarly, $\sqrt{x^2} = x$ since $x \times x = x^{1+1} = x^2$,

and $\sqrt{x^4} = x^2$ since $x^2 \times x^2 = x^{2+2} = x^4$.

Also, $\sqrt{x^6} = x^3$. Why?

$\sqrt{x^8} = x^4$. Why?

Study the pattern.

For solutions to the **Audio Activity**, turn to **Activity 2** in the **Appendix**, **Topic 1**.

Activity 2

Evaluate the square root of a perfect square.

To evaluate a square root, you must find a number (or variable) which, when multiplied by itself, produces the radicand.

For example, $\sqrt{4} = 2$ because the answer, 2, multiplied by itself, yields the radicand 4.

You can find the square root of any perfect square if you can find two identical factors which multiplied together give the original radicand.

For example,

$\sqrt{\frac{4}{9}} = \frac{2}{3}$ because $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

$\sqrt{y^2} = y$ because $y \times y = y^2$

$\sqrt{y^6} = y^3$ because $y^3 \times y^3 = y^{3+3} = y^6$

Now try the following exercise.

1. Determine the value of each. Do a minimum of five.

   a. $\sqrt{49}$ b. $\sqrt{121}$

   c. $\sqrt{169}$ d. $\sqrt{a^2}$

   e. $\sqrt{x^8}$ f. $\sqrt{z^6}$

   g. $\sqrt{64}$ h. $\sqrt{y^{16}}$

   i. $\sqrt{m^{12}}$

For solutions to **Activity 2**, turn to the **Appendix, Topic 1**.

Activity 3

Approximate the square root of a number, and check it by using a calculator.

What are you to do if you are asked to find the square root of a number like 6 which is not a perfect square? Try to guess the answer. You could write the following in order from smallest to largest:

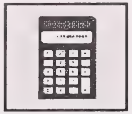
$\sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9}.$

Since $\sqrt{6}$ is between $\sqrt{4}$ and $\sqrt{9}$, it must be larger than $\sqrt{4}$ and smaller than $\sqrt{9}$.

$2 \times 2 = 4$, so $\sqrt{4} = 2$.

Since $\sqrt{4} = 2$ and $\sqrt{9} = 3$, then $\sqrt{6}$ must be larger than 2 and smaller than 3.

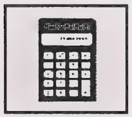
Try to find the answer to one decimal place.



$2.1 \times 2.1 = 4.41$
$2.2 \times 2.2 = 4.84$
$2.3 \times 2.3 = 5.29$
$2.4 \times 2.4 = 5.76$
$2.5 \times 2.5 = 6.25$
$2.6 \times 2.6 = 6.76$

From the information above, the answer to $\sqrt{6}$ must be somewhere between 2.4 and 2.5. You could continue guessing like this, and you could keep getting closer to $\sqrt{6}$. In reality, you will never get an exact answer because there is no rational number which is equal to $\sqrt{6}$.

You can show this by using a calculator.



Enter	Display
$\boxed{C}$	0
6	6
$\boxed{\sqrt{\ }}$	2.449489743

The display should read 2.449489743 (or to fewer decimal places depending on your calculator).

What happens if you multiply this number by itself?

Enter	Display
$\boxed{C}$	0
6	6
$\boxed{\sqrt{\ }}$	2.449489743
$\boxed{\times}$	2.449489743
$\boxed{=}$	6

Does this mean $(2.449\,489\,743)^2 = 6$? The answer is no. Your calculator may be sophisticated enough to actually have multiplied $\sqrt{6}$ by itself rather than the number 2.449 489 743. Now prove this.

Enter	Display
$\boxed{C}$	0
2.449489743	2.449489743
$\boxed{\times}$	2.449489743
$\boxed{=}$	6.000000001

You will not get the number 6. Rather, you will get 6.000 000 001 or some similar number. It is not possible to write $\sqrt{6}$ as an exact decimal number.

Keep trying until you get "close enough."

Example 3

Estimate the value of $\sqrt{30}$ to one decimal place, and check your answer with a calculator.

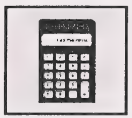
Solution:

$\sqrt{30}$ is between $\sqrt{25}$ and $\sqrt{36}$. (Notice you should always pick numbers that are perfect squares).

Therefore, $\sqrt{30}$ must be between 5 and 6. Write out a short table of possible one decimal answers.

$5.4 \times 5.4 = 29.16$
$5.5 \times 5.5 = 30.25$

Therefore, $\sqrt{30}$ must be between 5.4 and 5.5 (closer to 5.5). Check with your calculator.



Enter	Display
$\boxed{C}$	0
30	30
$\boxed{\sqrt{\ }}$	5.477225575

The answer 5.477 225 575 is between 5.4 and 5.5, so your approximation is correct.

Do at least questions 1, 2, and 3.

1. Give an approximation to two decimal places for each and check your answer with your calculator.

   a. $\sqrt{72}$ b. $\sqrt{3}$
   c. $\sqrt{150}$ d. $\sqrt{23}$
   e. $\sqrt{335}$

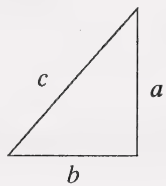
2. The time an object falls is related to the distance it falls by the following formula.

   $$t = \sqrt{\frac{2d}{9.8}}$$

   If an object falls a distance of 5 m, calculate the time it takes to fall. Give the answer to the nearest second.

3. The Pythagorean Theorem relates the lengths of the sides of a right-angled triangle as

   $c^2 = a^2 + b^2$, so $c = \sqrt{a^2 + b^2}$.

   Calculate the length of the hypotenuse, c, when $a = 3$ m and $b = 5$ m. Give your answer to one decimal place.

4. A manufacturer is asked to make square lids for boxes so that the area of each lid is 1.5 m^2. Find the lengths of the sides of each lid. Give your answer to three decimal places.

Notice the $\sqrt{\ }$ on a calculator returns the positive square root.

5. The formula for the area of a circle is $A = \pi r^2$, and r is the radius. Find the radius of a circle with $A = 10\ \text{m}^2$. Give the answer to two decimal places. (Recall: $\pi \doteq 3.141\,593$)

For solutions to **Activity 3**, turn to the **Appendix, Topic 1**.

You may study the next two activities by listening to the audiotape or by studying the print, or you may do both.

Audio Activity - Calculating Cube Roots

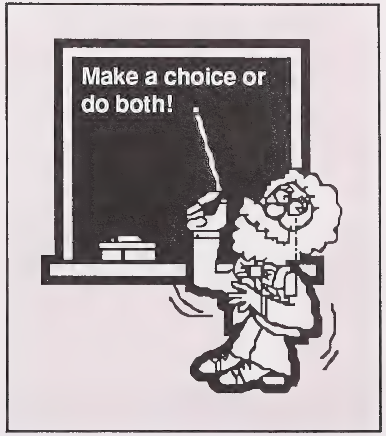
Activity 4

Evaluate the cube root of a perfect cube.

Activity 5

Approximate the cube root of a number, and check it by using a calculator.

Your tape *Mathematics 23 - Evaluating Radicals* should be cued and ready for this audio activity on calculating cube roots. After you have listened to the audiotape, complete the questions which follow the audio windows.



Calculating Cube Roots

1

Perfect Cubes

a. $\sqrt[3]{8} = ?$

b. $\sqrt[3]{27} = ?$

You know that $2 \times 2 \times 2 = 8$, ____ × ____ × ____ = 27.
(Try 3)

So, $\sqrt[3]{8} = 2$,

$$\underbrace{\underbrace{3 \times 3}_{9} \times 3}_{27} = 27$$

and 8 is a perfect cube.

2

Cubes and Cube Roots

n	n^3
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

3

$$(-1)(-1)(-1) = -1$$

$$\therefore \sqrt[3]{-1} = -1$$

Also $\sqrt[3]{-27} = -3$

since $(-3)(-3)(-3) = -27$.

also $\sqrt[3]{-y^9} = -y^3$

since $\left(-y^3\right)\left(-y^3\right)\left(-y^3\right) = -y^9$.

4

Approximating Cube Roots

What is the cube root of 156?

Because 156 lies between 125 and 216, $\sqrt[3]{156}$ lies between 5 and 6.

5

Finding Cube Roots

Using the calculator (use the inverse and x^y keys), find $\sqrt[3]{156}$.

Enter 156

Press $\boxed{\text{INV}}$

Press $\boxed{x^y}$

Enter 3

Press $\boxed{=}$

Display = 5.383212612 or 5.4 (to one decimal place).

6 Find the Cube Roots to the Nearest Whole Number

1. $\sqrt[3]{27} =$ ______	2. $\sqrt[3]{216} =$ ______	3. $\sqrt[3]{729} =$ ______
4. $\sqrt[3]{64} =$ ______	5. $\sqrt[3]{343} =$ ______	6. $\sqrt[3]{x^6} =$ ______
7. $\sqrt[3]{125} =$ ______	8. $\sqrt[3]{512} =$ ______	9. $\sqrt[3]{a^{12}} =$ ______

7 Cube Roots of Variable Expressions

You know $\sqrt[3]{27} = 3$ since $3 \times 3 \times 3 = 27$.

Also, $\sqrt[3]{x^3} = x$ since $x \times x \times x = x^{1+1+1} = x^3$.

And, $\sqrt[3]{x^6} = x^2$ since $x^2 \times x^2 \times x^2 = x^{2+2+2} = x^6$.

Similarly, $\sqrt[3]{8a^9} = 2a^3$ since
$$\begin{aligned} 2a^3 \times 2a^3 \times 2a^3 &= 2^{1+1+1} \times a^{3+3+3} \\ &= 2^3 \times a^9 \\ &= 8a^9. \end{aligned}$$

For solutions to the **Audio Activity**, turn to **Activity 4** in the **Appendix, Topic 1**.

Activity 4

Evaluate the cube root of a perfect cube.

To evaluate a cube root, you must find a number (or variable) which, when multiplied by itself three times, produces the radicand.

Example 4

Find $\sqrt[3]{8}$.

Solution:

Since $2\times2\times2=8$, then $\sqrt[3]{8}=2$.

Example 5

Find $\sqrt[3]{y^9}$.

Solution:

Since $y^3\times y^3\times y^3=y^{3+3+3}=y^9$, then $\sqrt[3]{y^9}=y^3$.

1. Determine the value of each of the following.

a. $\sqrt[3]{27}$ b. $\sqrt[3]{729}$ c. $\sqrt[3]{343}$

d. $\sqrt[3]{x^6}$ e. $\sqrt[3]{125}$ f. $\sqrt[3]{a^{12}}$

For solutions to **Activity 4**, turn to the **Appendix, Topic 1**.

Consider the following product:

$(-1)(-1)(-1)=-1$

From the above, it seems that if you take the negative of any cube root and cube it, you get a negative value.

In general, when you take the cube root of any negative value, you get a negative value.

Example 6

Find $\sqrt[3]{-27}$.

Solution:

$\sqrt[3]{-27}=-3$ because $(-3)(-3)(-3)=-27$.

Example 7

Find $\sqrt[3]{-y^9}$.

Solution:

$\sqrt[3]{-y^9}=-y^3$ because $(-y^3)(-y^3)(-y^3)=-y^9$.

Do any three of the following questions.

2. Determine the simplest solution to each.

a. $\sqrt[3]{-64}$ b. $\sqrt[3]{-y^6}$ c. $\sqrt[3]{\frac{-27}{64}}$

d. $\sqrt[3]{-216}$ e. $\sqrt[3]{-125}$

For solutions to **Activity 4**, turn to the **Appendix, Topic 1**.

Activity 5

Approximate the cube root of a number, and check it by using a calculator.

This table of cubes and cube roots will come in handy for making approximations to cube roots.

$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$
$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^3 = 512$	$\sqrt[3]{512} = 8$
$9^3 = 729$	$\sqrt[3]{729} = 9$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$

Example 8

Give an approximate value of $\sqrt[3]{51}$.

Solution:

Use your table. You find $\sqrt[3]{51}$ lies between the cube roots of two perfect cubes; $\sqrt[3]{27}$, $\sqrt[3]{51}$, $\sqrt[3]{64}$.

Therefore, $\sqrt[3]{51}$ must be between 3 and 4 because $\sqrt[3]{27} = 3$ and $\sqrt[3]{64} = 4$.

You can check your answer by using a calculator.

Enter	Display
C	0
51	51
x^y	51
3	3
=	3.708429769

You get $\sqrt[3]{51} = 3.708\,429\,769$. Then $\sqrt[3]{51} = 3.71$ (to two decimal places).

Do question 1, and either question 2 or 3. Do the remaining question if you need more practice.

1. Give an approximate answer to each; then check by using your calculator.

   a. $\sqrt[3]{360}$ b. $\sqrt[3]{90}$ c. $\sqrt[3]{-20}$

2. The volume of a cubical box is given by $V = s^3$, where s is the length of a side. The length of a side is given by $s = \sqrt[3]{V}$. Calculate the length of a side of a cubical box with a volume of 3 m^3. Give the answer to two decimal places.

3. The radius of a sphere is given by $\sqrt[3]{\dfrac{3V}{4\pi}}$, where V is the volume. What is the radius of a basketball with volume 0.113 m^3? Give the answer to two decimal places.

For solutions to **Activity 5**, turn to the **Appendix, Topic 1**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

You may decide to do both.

Extra Help

What Is A Radical?

A **radical expression** (or a **radical**) is an expression involving a root sign. Two examples of radicals are $\sqrt{77}$ and $\sqrt[3]{85}$. The value under the root sign is called the **radicand**. The number written in the "crook" of the radical sign is called the **index**. When a radical has no written index, it is understood to be 2.

A **second order radical** (commonly called a square root) involves a radical sign with an understood index of 2. Two examples of second order radicals are $\sqrt{x}$ and $\sqrt{10}$. $\sqrt{x}$ has the radicand x, and $\sqrt{10}$ has the radicand of 10.

A **third order radical** (commonly called a cube root) involves a radical sign with an index of 3. Two examples of third order radicals are $\sqrt[3]{792}$ and $\sqrt[3]{x}$. $\sqrt[3]{792}$ has the radicand 792, and $\sqrt[3]{x}$ has the radicand x.

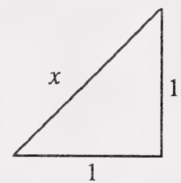
Second Order Radicals

Some second order radicals that often occur in mathematics are $\sqrt{2}, \sqrt{3}$, and $\sqrt{7}$. You must be able to handle such values.

Square roots can sometimes be used to represent the length of a segment.

Example 9

Suppose you need to find the length of the hypotenuse of the right-angled triangle shown below.



Solution:

By the Theorem of Pythagoras you know the square on the hypotenuse is equal to the sum of the squares on the other two sides.

$$\begin{aligned} x^2 &= 1^2 + 1^2 \\ &= 1 + 1 \\ &= 2 \\ x &= \sqrt{2} \\ &\doteq 1.414 \end{aligned}$$

The length of the hypotenuse is approximately 1.414 units, and the radical $\sqrt{2}$ can represent the length of the hypotenuse segment.

Perfect Squares

A **perfect square** is a number which can be written as a power with an exponent of 2.

Numbers such as 4, 25, 49, and 100 are examples of perfect squares.

The perfect square of 64 could be written as 8^2 or as $(-8)^2$.

Square Roots

The **square root** of a number is the base when the number is written as a power with an exponent of 2. Thus, 8 and – 8 are the square roots of 64.

Every positive real number has two square roots that are equal in absolute value but opposite in sign.

The positive square root of a number is called its **principal square root.**

When you write the radical sign $\sqrt{}$, you mean "square root." Whenever you use the radical sign, you mean the positive (or principal) square root.

Thus, $\sqrt{64} = 8$, and $\sqrt{81} = 9$.

Consider the radical $\sqrt{7}$.

radical sign $\rightarrow \sqrt{7} \leftarrow$ radicand

$\sqrt{7}$ can be classified as an **entire** second order radical. Some other entire second order radicals are $\sqrt{11}$, $\sqrt{17}$, and $\sqrt{23}$.

A **mixed radical** involves the product of a rational number and a radical. For example, $3\sqrt{5}$ is a mixed radical.

rational number $\rightarrow \underbrace{3\sqrt{5}}_{\text{mixed radical}}$ $\leftarrow$ radical sign

Some other mixed radicals are $-7\sqrt{2}$, $\frac{3}{4}\sqrt{5}$, $0.5\sqrt{3}$.

Now try the following questions.

1. Write entire second order radicals using the following radicands.

   a. 7   b. 102   c. x d. $\frac{-2}{3}$

   e. 0.35   f. 57   g. 0.9

2. Give the principal square root of each of the following second order radicals.

   a. $\sqrt{100}$ b. $\sqrt{49}$ c. $\sqrt{1600}$

   d. $\sqrt{144}$ e. $\sqrt{1}$ f. $\sqrt{900}$

   g. $\sqrt{169}$ h. $\sqrt{0.04}$ i. $\sqrt{0.81}$

   j. $\sqrt{0.01}$

For solutions to **Extra Help,** turn to the **Appendix, Topic 1.**

Extensions

The square root function of your calculator is a specialized function from a more general function $\boxed{y^x}$ or $\boxed{x^y}$.

Try this sequence:

Enter	Display
$\boxed{C}$	0
4	4
$\boxed{x^y}$	4
2	2
$\boxed{=}$	16

You should get 16 on your display, which indicates you have entered $4^2 = 16$.
On the calculator, the inverse of x^y is $x^{\frac{1}{y}}$. Check to see what you get using the following procedure.

Enter	Display
$\boxed{C}$	0
4	4
$\boxed{INV}$	4
$\boxed{x^y}$	4
2	2
$\boxed{=}$	2

You should get 2 on your display, indicating $4^{\frac{1}{2}} = 2$. You know $\sqrt{4} = 2$, thus implying $4^{\frac{1}{2}} = \sqrt{4}$.

Try finding the square root of some other numbers to convince yourself that this is indeed another way to calculate square roots.

Do at least five of the following eight question parts. Do the remaining parts if you need additional practice.

1. Use your calculator and the function $\boxed{x^{\frac{1}{y}}}$ to calculate the following. Use the $\boxed{\sqrt{\ }}$ to check your answers.

Give your answers to two decimal places.

a. $\sqrt{79}$ b. $\sqrt{5}$

c. $\sqrt{125}$ d. $\sqrt{60}$

e. $\sqrt{400}$ f. $\sqrt{0.64}$

g. $\sqrt{0.04}$ h. $\sqrt{1000}$

For solutions to **Extensions,** turn to the **Appendix, Topic 1**.

Topic 2 Rational Exponents

Introduction

In this topic, you will investigate the relationship between terms with fractional exponents and the radical expressions of the previous topic.
For example, what does the expression $27^{\frac{2}{3}}$ mean, and to what simpler value can you reduce it?

What Lies Ahead

Throughout the topic you will learn to

1. transform expressions from radical to exponential form and vice versa

2. simplify and evaluate radical and exponential expressions

Now that you know what to expect, turn the page to begin your study of rational exponents.

Exploring Topic 2

Activity 1

Transform expressions from radical to exponential form and vice versa.

Consider a pattern of numbers.

$$2^2 \times 2^2 = 2^{2+2}$$
$$= 2^4$$

Therefore, $\sqrt{2^4} = 2^2$.

$$2^3 \times 2^3 = 2^{3+3}$$
$$= 2^6$$

Therefore, $\sqrt{2^6} = 2^3$.

$$2^{\frac{1}{2}} \times 2^{\frac{1}{2}} = 2^{\frac{1}{2}+\frac{1}{2}}$$
$$= 2^1$$

Therefore, $\sqrt{2^1} = 2^{\frac{1}{2}}$.

Can you see what pattern has been produced? In other words, since you can use the product law for exponents, you can write the square root of any number as that number raised to the power one-half.

In general, $x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^{\frac{1}{2}+\frac{1}{2}}$
$$= x^1$$
$$= x.$$

Therefore, $\sqrt{x} = x^{\frac{1}{2}}$.

Consider another pattern.

$$2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} = 2^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$$
$$= 2^1$$
$$= 2.$$

Therefore, $\sqrt[3]{2} = 2^{\frac{1}{3}}$.

In general, $x^{\frac{1}{3}} \times x^{\frac{1}{3}} \times x^{\frac{1}{3}} = x^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$
$$= x.$$

Therefore, $\sqrt[3]{x} = x^{\frac{1}{3}}$.

You can take specific examples and expand them in this way.

You can obtain the overall general equation
$x^{\frac{1}{b}} = \sqrt[b]{x}$.

You read $\sqrt[b]{x}$ as the b^{th} root of x.

Example 1

Convert $\sqrt[5]{3}$ to exponential form.

Solution:

$\sqrt[5]{3} = 3^{\frac{1}{5}}$

Example 2

Convert $y^{\frac{1}{7}}$ to radical form.

Solution:

$y^{\frac{1}{7}} = \sqrt[7]{y}$

Do a minimum of two from each of questions 1 and 2.

1. Convert the following to exponential form.

   a. $\sqrt[6]{4}$ b. $\sqrt[x]{8}$

   c. $\sqrt[9]{a}$ d. $\sqrt[4]{5}$

2. Convert these to radical form.

   a. $y^{\frac{1}{4}}$ b. $3^{\frac{1}{y}}$

   c. $7^{\frac{1}{2}}$ d. $x^{\frac{1}{3}}$

For solutions to **Activity 1**, turn to the **Appendix, Topic 2**.

Now consider the number $\sqrt[3]{4^2}$, and apply the general rule to this number.

You get $\sqrt[3]{4^2} = \left(4^2\right)^{\frac{1}{3}}$.

You can now use the Power of a Power Law to write $\left(4^2\right)^{\frac{1}{3}} = 4^{2\times\frac{1}{3}}$

$= 4^{\frac{2}{3}}$.

In general, $x^{\frac{a}{b}} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$.

Example 3

Write $\sqrt[3]{9^4}$ in exponential form.

Solution:

$\sqrt[3]{9^4} = 9^{\frac{4}{3}}$

Example 4

Write $a^{\frac{x}{2}}$ in radical form.

Solution:

$a^{\frac{x}{2}} = \sqrt[2]{a^x}$

$= \sqrt{a^x}$

Do a minimum of two from each of questions 3 and 4. Do the remaining questions if you need additional practice.

3. Write the following in exponential form.

   a. $\sqrt[5]{2^3}$ b. $\sqrt[y]{3^4}$

   c. $\sqrt[4]{y^9}$ d. $\left(\sqrt[3]{4}\right)^2$

4. Write these in radical form.

   a. $3^{\frac{2}{3}}$ b. $9^{\frac{4}{5}}$

   c. $x^{\frac{3}{y}}$ d. $4^{\frac{3}{4}}$

For solutions to **Activity 1**, turn to the **Appendix, Topic 2**.

Activity 2

Simplify and evaluate radical and exponential expressions.

You can sometimes evaluate an expression or simplify an expression using the power laws and the skills learned in this unit.

Look at the expression $64^{\frac{2}{3}}$.

You can solve this expression without a calculator! First write it in a different form.

$$64^{\frac{2}{3}} = \sqrt[3]{64^2}$$
$$= \left(\sqrt[3]{64}\right)^2$$

Now $\sqrt[3]{64} = 4$, since $4 \times 4 \times 4 = 64$.

Therefore, $\left(\sqrt[3]{64}\right)^2 = (4)^2$
$= 16$.

Another way to solve the expression is as follows.

$$64^{\frac{2}{3}} = \left(\sqrt[3]{64}\right)^2$$
$$= \left(\sqrt[3]{4^3}\right)^2$$
$$= \left(4^{\frac{3}{3}}\right)^2$$
$$= \left(4^1\right)^2$$
$$= 4^2$$
$$= 16$$

Do at least six of the following. If you need additional practice, do the rest of the question parts.

1. Simplify the following expressions completely.

a. $81^{\frac{3}{4}}$

b. $27^{\frac{4}{3}}$

c. $32^{\frac{6}{5}}$

d. $125^{\frac{2}{3}}$

e. $\left(\frac{9}{16}\right)^{\frac{1}{2}}$

f. $(-27)^{\frac{2}{3}}$

g. $(36)^{\frac{3}{2}}$

h. $\left(\sqrt[4]{16}\right)^3$

i. $\sqrt[3]{64^4}$

j. $\dfrac{1}{8^{\frac{2}{3}}}$

k. $81^{\frac{3}{2}}$

For solutions to **Activity 2**, turn to the **Appendix, Topic 2.**

If you have expressions which involve negative exponents, you use the power laws to change them to positive exponents before solving.

$a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-m}} = a^m$, where $a \neq 0$.

Example 5

Simplify $27^{\frac{-2}{3}}$.

Solution:

$$\begin{aligned} 27^{\frac{-2}{3}} &= \frac{1}{27^{\frac{2}{3}}} \\ &= \frac{1}{\left(\sqrt[3]{27}\right)^2} \\ &= \frac{1}{3^2} \\ &= \frac{1}{9} \end{aligned}$$

Example 6

Simplify $\sqrt[3]{(-64)^{-5}}$.

Solution:

$$\begin{aligned} \sqrt[3]{(-64)^{-5}} &= \left(\sqrt[3]{-64}\right)^{-5} \\ &= \frac{1}{\left(\sqrt[3]{-64}\right)^5} \\ &= \frac{1}{(-4)^5} \\ &= \frac{1}{-1024} \\ &= -\frac{1}{1024} \end{aligned}$$

Example 7

Simplify $\frac{1}{4^{-2}}$.

Solution:

$$\begin{aligned} \frac{1}{4^{-2}} &= 4^2 \\ &= 16 \end{aligned}$$

Do at least six of the following question parts.

2. Simplify the following expressions .

   a. $8^{\frac{-1}{3}}$ b. $(-1)^{\frac{1}{3}}$

   c. $(-1)^{\frac{-1}{3}}$ d. $(-1)^{\frac{-2}{5}}$

   e. $32^{\frac{-2}{5}}$ f. $\left(\frac{16}{25}\right)^{\frac{-1}{2}}$

   g. $\sqrt{x^{-4}}$ h. $\left(y^6\right)^{\frac{-1}{3}}$

   i. $\frac{1}{x^{-3}}$ j. $\frac{1}{3^{-2}}$

For solutions to **Activity 2**, turn to the **Appendix**, **Topic 2**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

If you have trouble changing expressions in exponential form to radical form, the reason may be that you don't "see" the relationship between the position of numbers and the meaning. Look closely at a variety of expressions, and note carefully the position and meaning of the numbers.

$$2^{\frac{1}{3}} = \sqrt[3]{2^1}$$

Do you see how the numbers are positioned?

The 2 is the base and goes under the radical sign.

The 1 is the numerator of the fractional exponent and goes to the exponent position of the base.

The 3 is the denominator of the fractional exponent and goes into the elbow of the radical sign.

Changing from a radical expression to an exponential form follows the same pattern in reverse.

$$\sqrt[3]{2^1} = 2^{\frac{1}{3}}$$

In the example in the previous column, the 1 has been put in on purpose to show you that even though it may not be included in an expression, you must understand that the 1 is implied. The following is the same example leaving out the 1 where it is permissible.

$$2^{\frac{1}{3}} = \sqrt[3]{2}$$

$$\text{and } \sqrt[3]{2} = 2^{\frac{1}{3}}$$

There is another number that can be left out of radical expressions which can cause confusion. Look at this example.

$$\sqrt{3^3} = 3^{\frac{3}{2}}$$

Notice the 2 is missing in the radical expression.

$$\sqrt[2]{} = \sqrt{}$$

It would be written like $\sqrt[2]{3^3} = 3^{\frac{3}{2}}$, but the 2 is usually omitted in the radical sign.

Example 8

Write $\sqrt{3}$ as an exponential expression.

Solution:

$\sqrt{3} = 3^{\frac{1}{2}}$

Notice both numbers 1 and 2 were left out of the radical expression but must be included in the exponential expression.

Take a look at the meaning of the 3 in the expression $\sqrt[3]{2}$.

The 3 in the position $\sqrt[3]{\ }$ means the cube root, or what number multiplied by itself three times gives the number under the radical sign (in this case, 2).

Do these in order.

- What number multiplied by itself three times gives x?

The solution is $\sqrt[3]{x}$.

This means $\sqrt[3]{x} \times \sqrt[3]{x} \times \sqrt[3]{x} = x$.

- What number multiplied by itself four times gives x?

The solution is $\sqrt[4]{x}$.

This means $\sqrt[4]{x} \times \sqrt[4]{x} \times \sqrt[4]{x} \times \sqrt[4]{x} = x$.

- What number multiplied by itself five times gives x?

The solution is $\sqrt[5]{x}$.

This means $\sqrt[5]{x} \times \sqrt[5]{x} \times \sqrt[5]{x} \times \sqrt[5]{x} \times \sqrt[5]{x} = x$.

Since $\sqrt[3]{x} = x^{\frac{1}{3}}$, then the 3 in the exponential expression must mean the same thing as the 3 in the radical expression.

This may be shown in a similar manner.

What number multiplied by itself three times gives x?

The solution is $x^{\frac{1}{3}}$.

This means $x^{\frac{1}{3}} \times x^{\frac{1}{3}} \times x^{\frac{1}{3}} = x$.

- What number multiplied by itself four times gives x?

The solution is $x^{\frac{1}{4}}$.

This means $x^{\frac{1}{4}} \times x^{\frac{1}{4}} \times x^{\frac{1}{4}} \times x^{\frac{1}{4}} = x$.

- What number multiplied by itself five times gives x?

The solution is $x^{\frac{1}{5}}$.

This means $x^{\frac{1}{5}} \times x^{\frac{1}{5}} \times x^{\frac{1}{5}} \times x^{\frac{1}{5}} \times x^{\frac{1}{5}} = x$.

A more complicated situation does not change the meaning that has been shown in previous examples.

For example, change $\sqrt[3]{8^2}$ to an exponential expression.

$$\sqrt[3]{8^2} = 8^{\frac{2}{3}}$$

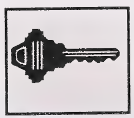
This looks exactly like the situation on the previous page where you have $\sqrt[3]{2^1}$.

The meaning of the 2 in the expression $\sqrt[3]{8^2}$ cannot be ignored in the way you ignored the meaning of the 1 in the expression $\sqrt[3]{2^1}$.

2^1 means 2, but 8^2 means multiply 8 by itself.

Another way of writing $\sqrt[3]{8^2}$ is $\left(\sqrt[3]{8}\right)^2$, so $\left(\sqrt[3]{8}\right)^2 = 8^{\frac{2}{3}}$.

This concludes the discussion of converting rational expressions to exponential expressions and vice versa. Now you should see how this general expression was obtained.



$$x^{\frac{a}{b}} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$$

Do as many as you feel are necessary to master the concepts.

1. Write each of the following as a radical expression.

   a. $2^{\frac{1}{2}}$ b. $3^{\frac{1}{4}}$

   c. $5^{\frac{1}{2}}$ d. $9^{\frac{2}{3}}$

   e. $27^{\frac{1}{3}}$ f. $16^{\frac{3}{4}}$

   g. $64^{\frac{5}{2}}$ h. $x^{\frac{2}{5}}$

2. Write each of the following in exponential form.

   a. $\sqrt{8}$ b. $\sqrt[3]{3}$

   c. $\left(\sqrt{9}\right)^3$ d. $\sqrt{25^3}$

   e. $\sqrt[4]{x^3}$ f. $\left(\sqrt[3]{4}\right)^2$

   g. $\sqrt[5]{7^4}$ h. $\sqrt[6]{12^3}$

For solutions to **Extra Help**, turn to the **Appendix, Topic 2.**

Extensions

Calculate $\sqrt{0.81}$. You get 0.9. There seems to be a common link between these numbers compared to $\sqrt{81} = 9$.
Try to determine the link by calculating the square roots of the following:

0.64, 0.49, 0.36, 0.25, 0.16, 0.9, 0.4

You will notice that the numbers decrease until you get to 0.9 and that the square roots work out evenly until you get to 0.9. What went wrong? You can find out by calculating $(0.3)^2$. You get 0.09. This would then produce a descending sequence:

0.64, 0.49, 0.36, 0.25, 0.16, 0.09, 0.04

Apply what you have seen to other numbers.

For example, without using your calculator, find $\sqrt{0.0081}$.

Since $\sqrt{81} = 9$ and $\sqrt{0.81} = 0.9$, it seems that as the number under the square root sign decreases by a divisor of 100, the answer decreases by a divisor of 10.

So your answer to $\sqrt{0.0081}$ should be 0.09. Check your answer with a calculator.

What about $\sqrt{-4}$? Is there a number which when multiplied by itself equals -4? See what the calculator gets you.

Enter	Display
C	0
4	4
+/–	-4
$\sqrt{\ }$	-E-

The display reads -E-, indicating an error on your part or a calculation that is not allowed. In this case, both reasons are valid. The square root of a negative number cannot be found using the set of real numbers for possible answers. (There is another set of numbers that will allow such a thing, but you will have to wait for a more advanced course). Explore the reason behind this strange answer to $\sqrt{-4}$. You know $2 \times 2 = 4$, so try $(-2)(-2)$. This also equals 4, so you can never get $\sqrt{-4}$ to be a real number.

Do as many questions as necessary to ensure you have mastered the concepts.

1. Without using your calculator, determine the value of each.

   a. $\sqrt{0.0036}$ b. $\sqrt{0.000\,081}$

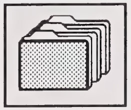
   c. $\sqrt{0.01}$ d. $\sqrt{0.0001}$

   e. $\sqrt{0.000\,001}$ f. $\sqrt{0.000\,000\,64}$

   g. $\sqrt{-25}$ h. $\sqrt{-0.0036}$

For solutions to **Extensions**, turn to the **Appendix**, **Topic 2**.

Unit Summary



What You Have Learned

In this unit you have learned

- the mathematical names of the parts of radical expressions; that is, index, radicand, radical sign, and root

- to evaluate square roots and cube roots such as

  $\sqrt{49} = 7$

  $\sqrt{x^6} = x^3$

  $\sqrt[3]{-8} = -2$

  $\sqrt[3]{y^9} = y^3$

- to transform expressions from radical to exponential form and vice versa

  $7^{\frac{3}{4}} = \sqrt[4]{7^3}$

  $\left(\sqrt[3]{6}\right)^2 = 6^{\frac{2}{3}}$

- to simplify and evaluate radical and exponential expressions

  $8^{\frac{1}{3}} = 2$

  $\sqrt[5]{32^2} = 2^2 = 4$

You are now ready to complete the **Unit Assignment**.

Appendix

Solutions

Review

Topic 1 Evaluating Radicals

Topic 2 Rational Exponents

Review

1. The base is 6, and the exponent is 3.

2. $5^3 = 5 \times 5 \times 5$
   $= 125$

3. a. 64 c. 81 d. 49 e. 25

4. a. 8 c. 9 d. 7 e. 5

5. a. $(m^2)(m^2)(m^3) = m^{2+2+3}$
   $= m^7$

   b. $a^6 \div a^2 = a^{6-2}$
   $= a^4$

   c. $(p^4)^3 = p^{4\times 3}$
   $= p^{12}$

6. a. $\dfrac{(4a^2b^3)^2}{a^5}$

   $= \dfrac{4^2 a^{2\times 2} b^{3\times 2}}{a^5}$

   $= \dfrac{16a^4b^6}{a^5}$

   $= 16a^{4-5}b^6$

   $= 16a^{-1}b^6$

   $= \dfrac{16b^6}{a}$

   b. $\dfrac{3x^{-2}y^3}{x^4y^2}$

   $= 3x^{-2-4}y^{3-2}$

   $= 3x^{-6}y^1$

   $= \dfrac{3y}{x^6}$

   c. $\dfrac{(5x)^0}{(3x)^{-3}}$

   $= \dfrac{1}{3^{-3}x^{-3}}$

   $= 3^3x^3$

   $= 27x^3$

Exploring Topic 1

Activity 1

Identify the radicand, index, radical sign, and root in radical expressions.

Audio Activity

9 Complete the Blanks

Expression	Exponent	Radicand	Index	Radical
$\sqrt[5]{7^3}$	**3**	$\mathbf{7^3}$	**5**	$\sqrt[5]{\mathbf{7^3}}$
$\sqrt[3]{9m^4}$	**1,* 4**	$\mathbf{9m^4}$	**3**	$\sqrt[3]{\mathbf{9m^4}}$

* The 9 has an implied exponent of 1.

1. a. 3 b. 2
   c. 4^2 d. $\sqrt[3]{4^2}$
2. 4 (because $4 \times 4 = 16$)
3. 2 (because $2 \times 2 \times 2 = 8$)
4. a. 1, 7 b. $2y^7$
   c. 4 d. $\sqrt[4]{2y^7}$
5. 8 (because $8 \times 8 = 64$)
6. 4 (because $4 \times 4 \times 4 = 64$)

Activity 2

Evaluate the square root of a perfect square.

Audio Activity

11 Find the Square Roots

$\sqrt{16} =$ **4** $\sqrt{29} =$ **5.4**

$\sqrt{121} =$ **11** $\sqrt{76} =$ **8.7**

1. a. 7 b. 11 c. 13
   d. a e. x^4 f. z^3
   g. 8 h. y^8 i. m^6

Activity 3

Approximate the square root of a number, and check it by using a calculator.

1. a. 8.49 b. 1.73 c. 12.25

   d. 4.80 e. 18.30

2. one second since $t=\sqrt{\frac{2(5)}{9.8}}=\sqrt{\frac{10}{9.8}}$

   $\doteq 1$

   The object takes one second to fall.

3. 5.8 m since $c=\sqrt{3^2+5^2}$

   $=\sqrt{9+25}$

   $=\sqrt{34}$

   $\doteq 5.8$

   The length of the hypotenuse is 5.8 m.

4. 1.225 m since $s^2 = A$

   $s=\sqrt{A}$

   $=\sqrt{1.5}$

   $\doteq 1.225\,\text{m}$

   The length of each side is 1.225 m.

5. 1.78 m since $r=\sqrt{\frac{A}{\pi}}$

   $=\sqrt{\frac{10}{\pi}}$

   $\doteq \sqrt{3.1831}$

   $\doteq 1.78\,\text{m}$

   The radius of the circle is 1.78 m.

Activity 4

Evaluate the cube root of a perfect cube.

Audio Activity

6 Find the Cube Roots to the Nearest Whole Number

1. $\sqrt[3]{27} =$ **3 (because 3 × 3 × 3 = 27)**
2. $\sqrt[3]{216} =$ **6**
3. $\sqrt[3]{729} =$ **9**
4. $\sqrt[3]{64} =$ **4**
5. $\sqrt[3]{343} =$ **7**
6. $\sqrt[3]{x^6} =$ $\boldsymbol{x^2}$
7. $\sqrt[3]{125} =$ **5**
8. $\sqrt[3]{512} =$ **8**
9. $\sqrt[3]{a^{12}} =$ $\boldsymbol{a^4}$

1. a. 3 b. 9 c. 7
   d. x^2 e. 5 f. a^4

2. a. -4 b. $-y^2$ c. $-\frac{3}{4}$
   d. -6 e. -5

Activity 5

Approximate the cube root of a number, and check it by using a calculator.

1. a. 7.11 b. 4.48 c. -2.71

2. $s = \sqrt[3]{3}$
   $\doteq 1.44$

   The length of the side is 1.44 m.

3. $r = \sqrt[3]{\dfrac{3(0.113)}{4\pi}}$
   $\doteq 0.30$

   The radius of the basketball is 0.30 m.

Extra Help

1. a. $\sqrt{7}$ b. $\sqrt{102}$ c. $\sqrt{x}$
   d. $\sqrt{\frac{-2}{3}}$ e. $\sqrt{0.35}$ f. $\sqrt{57}$
   g. $\sqrt{0.9}$

2. a. 10 b. 7 c. 40
   d. 12 e. 1 f. 30
   g. 13 h. 0.2 i. 0.9
   j. 0.1

Extensions

1. a. 8.89 b. 2.24 c. 11.18
   d. 7.75 e. 20 f. 0.8
   g. 0.2 h. 31.62

Exploring Topic 2

Activity 1

Transform expressions from radical to exponential form and vice versa.

1. a. $4^{\frac{1}{6}}$ b. $8^{\frac{1}{x}}$
   c. $a^{\frac{1}{9}}$ d. $5^{\frac{1}{4}}$

2. a. $\sqrt[4]{y}$ b. $\sqrt[y]{3}$
   c. $\sqrt{7}$ d. $\sqrt[3]{x}$

3. a. $2^{\frac{3}{5}}$ b. $3^{\frac{4}{y}}$
   c. $y^{\frac{9}{4}}$ d. $4^{\frac{2}{3}}$

4. a. $\sqrt[3]{3^2}$ b. $\sqrt[5]{9^4}$
   c. $\sqrt[y]{x^3}$ d. $\sqrt[4]{4^3}$

Activity 2

Simplify and evaluate radical and exponential expressions.

1. a. 27 b. 81
   c. 64 d. 25
   e. $\frac{3}{4}$ f. 9
   g. 216 h. 8
   i. 256 j. $\frac{1}{4}$
   k. 729

2. a. $8^{\frac{-1}{3}} = \frac{1}{8^{\frac{1}{3}}}$
   $= \frac{1}{\sqrt[3]{8}}$
   $= \frac{1}{2}$

   b. $(-1)^{\frac{1}{3}} = \sqrt[3]{-1}$
   $= -1$

c. $(-1)^{\frac{-1}{3}} = \frac{1}{(-1)^{\frac{1}{3}}}$
$= \frac{1}{\sqrt[3]{-1}}$
$= \frac{1}{-1}$
$= -1$

d. $(-1)^{\frac{-2}{3}} = \frac{1}{(-1)^{\frac{2}{3}}}$
$= \frac{1}{\left(\sqrt[3]{-1}\right)^2}$
$= \frac{1}{(-1)^2}$
$= \frac{1}{1}$
$= 1$

e. $32^{\frac{-2}{5}} = \frac{1}{32^{\frac{2}{5}}}$
$= \frac{1}{\left(\sqrt[5]{32}\right)^2}$
$= \frac{1}{2^2}$
$= \frac{1}{4}$

f. $\left(\frac{16}{25}\right)^{\frac{-1}{2}} = \left(\frac{25}{16}\right)^{\frac{1}{2}}$
$= \sqrt{\frac{25}{16}}$
$= \frac{5}{4}$

g. $\sqrt{x^{-4}} = \sqrt{\frac{1}{x^4}}$
$= \frac{1}{x^2}$

h. $\left(y^6\right)^{\frac{-1}{3}} = y^{6 \times \frac{-1}{3}}$
$= y^{-2}$
$= \frac{1}{y^2}$

i. $\frac{1}{x^{-3}} = x^3$

j. $\frac{1}{3^{-2}} = 3^2$
$= 9$

Extra Help

1. a. $\sqrt{2}$ b. $\sqrt[4]{3}$
   c. $\sqrt{5}$ d. $\sqrt[3]{9^2}$
   e. $\sqrt[3]{27}$ f. $\sqrt[4]{16^3}$
   g. $\sqrt{64^5}$ h. $\sqrt[5]{x^2}$

2. a. $8^{\frac{1}{2}}$ b. $3^{\frac{1}{3}}$
   c. $9^{\frac{3}{2}}$ d. $25^{\frac{3}{2}}$
   e. $x^{\frac{3}{4}}$ f. $4^{\frac{2}{3}}$
   g. $7^{\frac{4}{5}}$ h. $12^{\frac{3}{6}} = 12^{\frac{1}{2}}$

Extensions

1. a. 0.06 b. 0.009
   c. 0.1 d. 0.01
   e. 0.001 f. 0.0008
   g. no possible real number answer
   h. no possible real number answer

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Mathematics 23

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